IMC Algebra

1. [IMC 2009 Q24] What is the largest number of the following statements that can be true at the same time?

$$0 < x^2 < 1, \qquad x^2 > 1, \qquad -1 < x < 0, \qquad 0 < x < 1, \qquad 0 < x - x^2 < 1$$

   A 1   B 2   C 3   D 4   E 5

2. [IMC 2008 Q19] Which of the following is equal to $(1 + x + y)^2 - (1 - x - y)^2$ for all values of x and y?

   A $4x$ B $2(x^2 + y^2)$ C 0 D $4xy$ E $4(x + y)$

3. [IMC 2006 Q18] Given that $4^x + 4^x + 4^x + 4^x = 4^{16}$, what is the value of x?

   A 2   B 4   C 8   D 12   E 15

4. [IMC 2006 Q20] Given that the number 2006 is the correct answer to the calculation
$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + (n - 2) - (n - 1) + n$$
   what is the sum of the digits of n?

   A 3   B 4   C 5   D 6   E 7

5. [IMC 2004 Q22] In a maths exam with N questions, you score m marks for a correct answer to each of the first q questions and $m + 2$ marks for a correct answer to each of the remaining questions. What is the maximum possible score?

   A $(m + 2)N - 2q$ B Nm C $mq + (m + 2)q$ D $N(m + 1)$
   E $Nm + q(m + 2)$

6. [IMC 2012 Q17] The first term of a sequence of positive integers is 6. The other terms in the sequence follow these rules:
   - if a term is even then divide it by 2 to obtain the next term
   - if a term is odd then multiply it by 5 and subtract 1 to obtain the next term.
   For which values of n is the nth term equal to n?

   A 10 only   B 13 only   C 16 only   D 10 and 13 only   E 13 and 16 only

Solutions:

1. C
2. E
3. E
4. D
5. A
6. E